In a nondissociating gas and assuming the medium in front of the shock at rest, Eq. (18) reduces to

$$t_c = -A \log\left(1 + \frac{A}{\lambda_0}\right)$$

which agrees with the expression obtained by Upadhyay.[6]

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Evidence of Imbedded Vortices in a Three-Dimensional Shear Flow

E. L. Morrisette* and D. M. Bushnell†

NASA Langley Research Center, Hampton, Va.

Introduction

THIS Note reports on flow phenomena encountered in an investigation originally conceived as a contribution to the experimental three-dimensional data base for use in turbulence modeling. These results suggest that turbulence modeling in three-dimensional mean flows may be considerably more difficult (compared to the two-dimensional case) than originally hoped, due to the possible existence of both steady and unsteady vortices and their interaction. The experiment was conducted in the Langley 20 in. Mach 6 Blowdown Air Tunnel[1] on the three-dimensional wedge model shown sketched on Fig. 1 (model run at zero pitch and yaw). The tip region was designed with a nearly constant surface pressure level and a nearly continuous surface curvature (Fig. 2) to avoid the formation of large tip vortices associated with pressure and surface curvature discontinuities.[2,3] The original object of the study was to measure the streamwise development of the three-dimensional wake region downstream of the tip region, where both $\partial u/\partial y$ and $\partial u/\partial z$ are present. The data were meant to represent a simple three-dimensional quasiparallel shear flow perhaps suitable to help begin the process of turbulence modeling in three-dimensional free flows (note pre-experiment idealization of expected flowfield shown on Fig. 1).

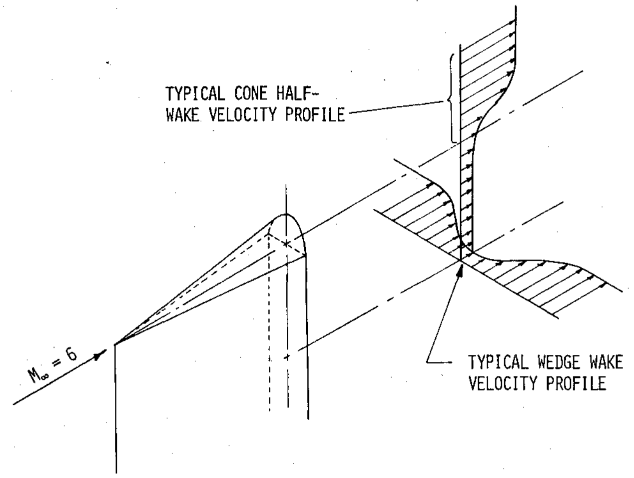

Fig. 1 Model and idealized velocity profiles in downstream wake.

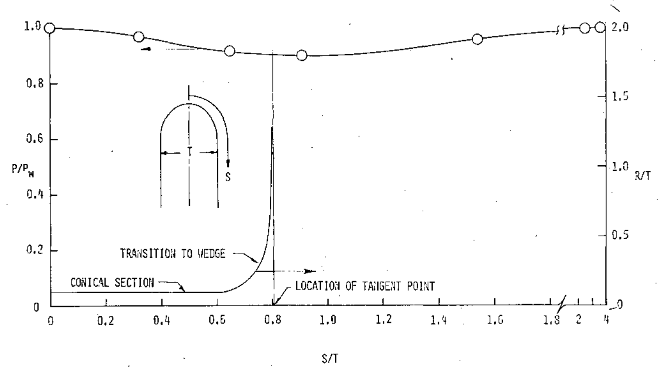

Fig. 2 Pressure distribution and radius of curvature around model near trailing edge.

Typical results of experimental pitot surveys are shown on Fig. 3. Instead of the rather simple flow sketched on Fig. 1, we obtained indication of a much more complex flow, with large deficits in longitudinal pitot pressure. Such deficits are usually associated with the core region of quasisteady longitudinal vortices. Vapor screen flow visualization results (Fig. 4) provide a further indication that the odd pitot distributions are indeed longitudinal vortices. These vortices 1) occur primarily in the tip region, becoming very weak as one approaches the two-dimensional zone away from the tip; and 2) evidently form in the vicinity of the wake neck. As an observation which may be of some technological importance, the overall wake thickness (and therefore turbulence entrainment rate) is increased by essentially 100%. (This suggests speculation that a fuel injector strut which is periodically necked down may provide somewhat faster mixing.) Previous work has already indicated that longitudinal vortices, once present, can considerably increase the mixing rate in free flows (e.g., "hypermixing" nozzles[4]). Trentacosta and Sforza[5] also found three-dimensional irregularities in the velocity profiles around the potential core of three-dimensional jets; however, their proposed explanation for the irregularities is the existence of a decaying ring vortex as opposing to the growing longitudinal vortices found in the present study.

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*Aerospace Engineer, Fluid Mechanics Branch, High-Speed Aerodynamics Division. Member AIAA.

†Head, Fluid Mechanics Branch, High-Speed Aerodynamics Division. Associate Fellow AIAA.

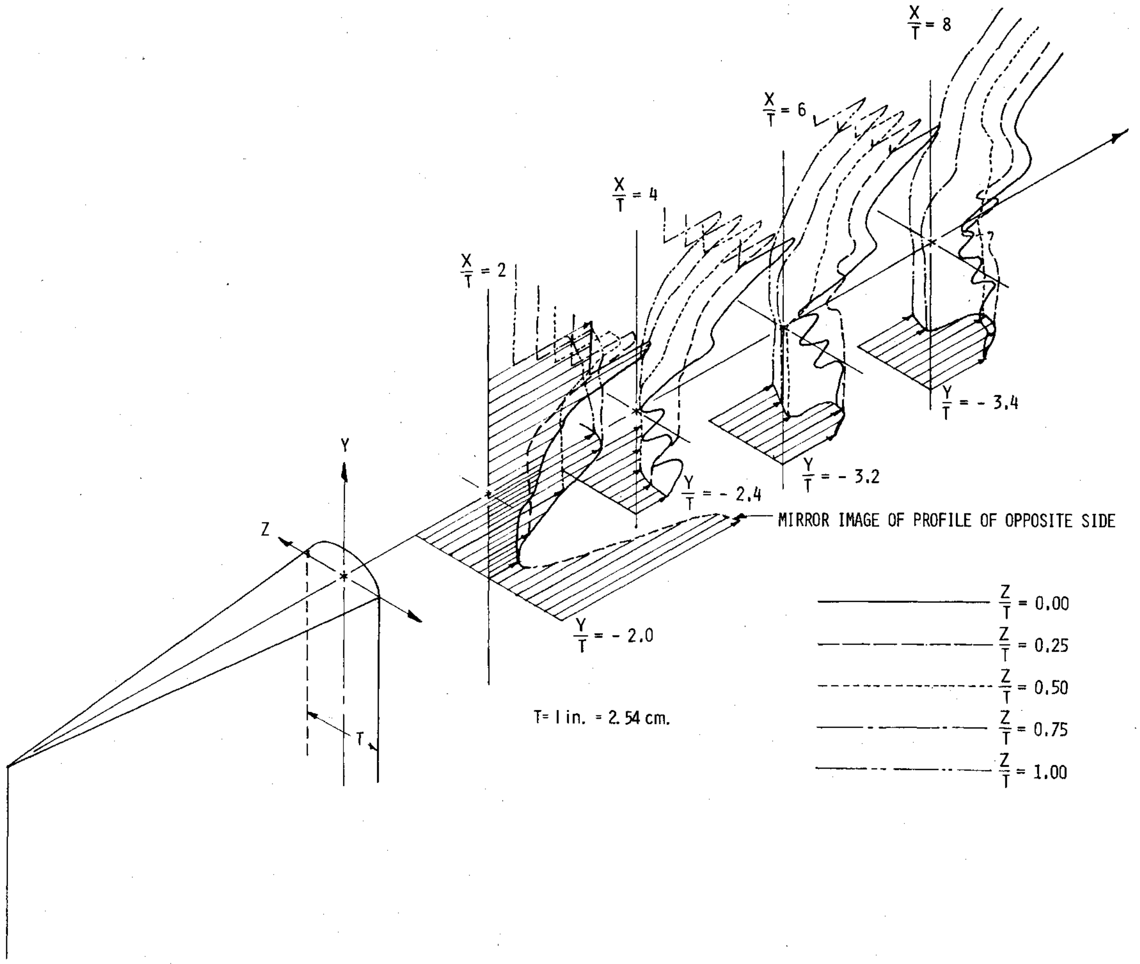

Fig. 3 Measured pitot pressure profiles in wake of body. Note that the Z direction is plotted on the opposite side of the axis from the true flow for a clearer plot, vertical (Y) scale is compressed to one-half that of the other two scales (X,Z).

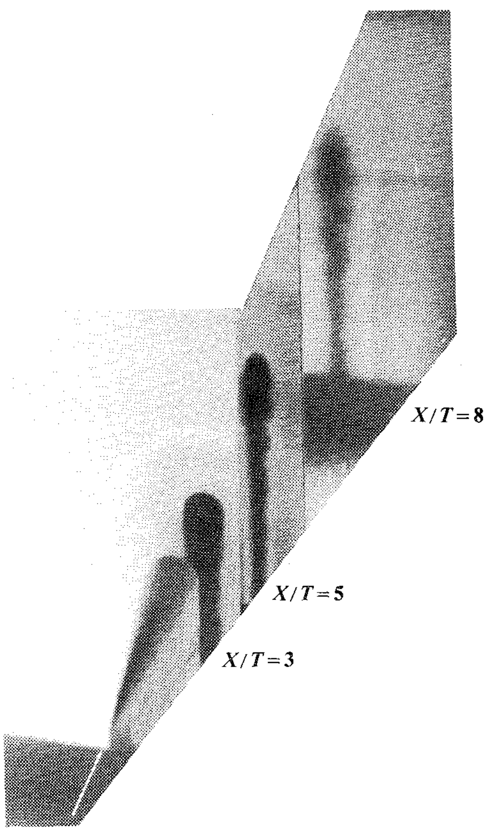

Fig. 4 Vapor screen pictures showing cross-section of the flow at three X stations.

The obvious question concerns the genesis of these vortices. One strong possibility is the generation of quasisteady Taylor-Gortler vortices in the concavely curved shear layer approaching the wake neck. In the somewhat analogous two-dimensional case studied heretofore of a shear layer approaching reattachment[6,7] such vortices were fairly weak and therefore would not generally cause large perturbations in the axial pitot pressure profile. In the present experiment the presence of the additional mean vorticity associated with the tip flow is evidently responsible for a large increase in vortex growth rate, in agreement with secondary flow production of Prandtl's first kind, derived from mean skewing.[8] Transition data on wedge models in the same tunnel indicate that transition should begin, but not be completed, on the surface of the present wake generator; therefore additional mean flow skewing, and possibly attendant higher vortex amplification, could arise from skewing of the transition region locus associated with the tip flow.

For three-dimensional shear flows, the existence of these longitudinal vortices and the possible existence of other vortex flows (e.g., Fig. 2.4.4 of Ref. 9) is another indication that turbulence modeling may be much more difficult than in the two-dimensional case. This is doubly unfortunate in that the experimental data in three-dimensional layers is considerably more difficult to obtain.

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Use of Constructed Variables in the Method of Strained Coordinates

Samuel C. McIntosh Jr.* and David Nixon†

Nielsen Engineering & Research, Inc., Mountain View, Calif.

In his book, Van Dyke[1] recommends that the method of strained coordinates, or the PLK method,[2] not be used with a "constructed" dependent variable, such as the velocity potential. The purpose of this Note is to clarify this recommendation and to demonstrate that a constructed dependent variable can be used, as long as the straining is determined by recourse to the proper physical variable.

As an example, consider the problem statement for supersonic flow past a thin airfoil (cf., Chap. 6 of Ref. 1). With the notation of Ref. 1, the problem statement for a uniform first approximation in terms of the velocity potential reads

$$\phi_{\xi\eta} + (\gamma+1)M^4\phi_\xi\phi_{\xi\xi}/2B^2 = 0 \tag{1}$$

Here M is the Mach number, $B=(M^2-1)^{1/2}$, and ξ and η "semicharacteristic" coordinates $x-By$ and By, respectively. The coordinate x is in the streamwise direction, and y is perpendicular to it. With the airfoil surface defined by $y=\epsilon T(x)$, and with ϵ a small parameter, the boundary conditions are given by

$$\phi_\xi(\xi,0) = -\epsilon T'(\xi)/B, \quad \phi=0 \text{ upstream} \tag{2}$$

A perturbation series is then defined as

$$\phi(\xi,\eta;\epsilon) \sim \epsilon\phi_1(s,t) + \epsilon^2\phi_2(s,t) + \ldots \tag{3}$$

and the coordinate straining is given by

$$\xi \sim s + \epsilon\xi_2(s,t) + \ldots, \quad \eta = t \tag{4}$$

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*Project Manager, Structures. Associate Fellow AIAA.

†Manager, CFD Department. Associate Fellow AIAA.

After substitution for $\phi(\xi,\eta;\epsilon)$ from Eq. (3) and use of Eqs. (4) to relate ξ and η derivatives to s and t derivatives, Eq. (1) becomes

$$\epsilon\phi_{1st} + \epsilon^2\phi_{2st} - \epsilon^2(\xi_{2s}\phi_{1st} + \xi_{2st}\phi_{1s} + \xi_{2t}\phi_{1ss}) + (\gamma+1)M^4\epsilon^2\phi_{1s}\phi_{1ss}/2B^2 = 0 \tag{5}$$

The first approximation is therefore given by solution of the equation

$$\phi_{1st} = 0 \tag{6}$$

with boundary conditions obtained from a similar treatment of Eqs. (2)

$$\phi_{1s}(s,0) = -T'(s)/B, \quad \phi_1 = 0 \text{ upstream} \tag{7}$$

The solution for ϕ_1 is

$$\phi_1(s,t) = -T(s)/B \tag{8}$$

and the differential equation for ϕ_2 becomes

$$\phi_{2st} = -\frac{1}{B}\frac{\partial}{\partial s}\left\{\xi_{2t}T'(s) + \frac{\gamma+1}{4}\frac{M^4}{B^3}[T'(s)]^2\right\} \tag{9}$$

If the straining rule is now applied to the velocity potential, ξ_2 can be chosen as so to eliminate the right-hand side of Eq. (9), thereby ensuring that ϕ_2 be no more singular than ϕ_1. This yields

$$\xi_{2t} = -(\gamma+1)M^4T'(s)/4B^3 \tag{10}$$

which is precisely half the "correct" straining that would be obtained if the problem were posed in terms of the velocity $u=\phi_\xi$ (Ref. 1).

It must be noted, however, that the straining rule in these two instances has been applied to different quantities. If the straining rule is applied by choosing ξ_2 so that the second approximation for the velocity is no more singular than the first, the results will not be the same. Integration of Eq. (9) yields

$$\phi_2(s,t) = -\{\xi_2T'(s) + (\gamma+1)M^4t[T'(s)]^2/4B^3\}/B + g(t) + k(s) \tag{11}$$

where $g(t)$ and $k(s)$ are functions of integration. With the velocity given as

$$u(\xi,\eta;\epsilon) = \phi_\xi = \epsilon u_1(s,t) + \epsilon^2u_2(s,t) + \ldots \tag{12}$$

the second approximation for the velocity is

$$u_2(s,t) = \phi_{2s} - \xi_{2s}\phi_{1s} \tag{13}$$

Use of Eqs. (8) and (11) for ϕ_1 and ϕ_2 yields

$$u_2(s,t) = -T''(s)[\xi_2 + (\gamma+1)M^4tT'(s)/2B^3]/B + k'(s) \tag{14}$$

If ξ_2 is now chosen so that u_2 is no more singular than u_1 as $t\to\infty$,

$$\xi_2(s,t) = -(\gamma+1)M^4tT'(s)/2B^3 + f(s) \tag{15}$$

and $f(s)=0$ so that the straining vanishes at $y=0$. Equation (15) reproduces Eq. (6.36) in Ref. 1, which is the "correct" straining. Hence the proper straining can be obtained as long as the straining rule is applied to the same quantity; whether a